CHAPTER 4

VARIOUS COSMOLOGICAL MODELS
THEIR TIME SCALES AND SPACE METRICS

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S U M M A R Y

Provisionally accepting the standard Robertson-Walker Metric (RWM) of cosmology and recalling that the principle of cosmic isotropy can be used as an argument for the definability of an all-embracing universal time, at least statistically, we propose to reverse this procedure by postulating such time as a regulative idea in the sense of Kant.

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1. INTRODUCTION

According to Milne & Walker, two kinds of observer-particles should be distinguished: *fundamental ones*, defining the structure of the specific cosmological model under consideration by *constituting its substratum*, and *accidental ones* which are *superposed upon the substratum.* The substratum thus serves as a universal "rest-frame", or "compass of inertia" (Weyl).

Taking the Robertson-Walker Metric (*RWM*) as our point of departure, we investigate the properties of two cosmological models: a) the *uniform expansion* model of Milne, which is the simplest model of a cosmic "big bang", and b) the *exponential expansion* model of Bondi & Gold, generally upposed to be the simplest possible model of a cosmic "steady state".

In accordance with a tentative acceptance of *RWM* we consider the relationship between our choice of time scale for a certain model of the universe and its corresponding space metric. As it turns out, there are at least two important ways of mapping the expansion of the universe: a) one which keeps atomic sizes constant while light is being stretched, and b) one which keeps distances between fundamental observers constant while their atoms are shrinking.

Finally, a new "steady state" model of the universe is proposed that deviates from *RWM* by allowing atoms to be contracted due to universal dispersion. In this model, spatial curvature is apparently increasing with the distance at which an object is seen by a fundamental observer. Constructed directly from $d\mathcal{T}^2 = dt^2 - ds^2 = dt^2\gamma^{-2}$ (*SR*), it is simpler than that of Bondi-Gold. The basic formal properties of this model and two related ones are then examined.

Added 2015:

As regards material properties it is obvious that, in a "steady state" universe, all constants of nature must be genuine constants relative to that time-scale which preserves the atomic radii. Therefore we cannot accept Milne's idea that Planck's constant varies relative to that scale.

By contrast, we are confronted with a choice as regards the validity of the law of inertia: does that law hold relative to the scale which preserves the atomic radii, or relative to that which preserves the distances between fundamental particles, thus leaving the substratum static?

Here we follow Milne, assuming the law of inertia to hold relative to the latter scale only. As a consequence, inertial motion relative to the latter scale must be described as accelerated relative to the former scale, i.e., that scale which preserves the atomic radii invariant.

This seems to open up the possibility for an explanation of the rotation pattern of galaxies and their clusters which makes the introduction of socalled "dark matter" redundant.

Added 2020:

The chapter has been abbreviated and simplified as compared to older versions.

Added 2021:

Some further simplifications have been made concerning the sections §§5&6.

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2. A CLASSICAL ALTERNATIVE TO SR

The hyperbolic formulae corresponding to the standard addition $\alpha' \equiv \alpha - \omega$ are:

$$cosh\alpha' = cosh\alpha\, cosh\omega - sinh\alpha\, sinh\omega$$
$$sinh\alpha' = sinh\alpha\, cosh\omega - cosh\alpha\, sinh\omega$$

Using $c_o \equiv unity$, $v \equiv tanh\omega$, the ***LT*** of ***SR*** with temporal coordinates become:

$$t' = t\, cosh\omega - x\, sinh\omega \;\; . \;\; x' = x\, cosh\omega - t\, sinh\omega$$

It is interesting that the *LT* are derivable from these addition formulae if and only if $\tau \equiv \tau'$ in:

$$t'/\tau' \equiv cosh\alpha' \;\; . \;\; t/\tau \equiv cosh\alpha \;\; . \;\; x'/\tau' \equiv sinh\alpha' \;\; . \;\; x/\tau \equiv sinh\alpha$$

It is natural to identify τ^2 with the ***SR*** invariant: $\mathcal{T}^2 \equiv t^2 - x^2 - y^2 - z^2 = t'^2 - x'^2 - y'^2 - z'^2$.

The standard (1x3) *LT* for three inertial frames, Σ, S & S', in relative motion are:

(1a) $\quad X \equiv x\, cosh\alpha - t\, sinh\alpha = x' cosh\alpha' - t' sinh\alpha' \;.\; Y \equiv y \equiv y'$

(1b) $\quad T \equiv t\, cosh\alpha - x\, sinh\alpha = t' cosh\alpha' - x' sinh\alpha' \;.\; Z \equiv z \equiv z'$

Consider Σ to be a preferred frame with the privileged observer Ω situated in its *origo*. Let the observers O & O' be situated in the *origos* of S & S', resp., and let the frame times t & t' of S & S' be synchronized to the proper time T of Ω by choosing $T \equiv t \equiv t' \equiv 0$ when O & O' both coincide with Ω. Suppose that an event E occurs at particle P, as observed by Ω, O & O'. Let the standard coordinates of E be (T, X, Y, Z) in Σ, (t, x, y, z) in S and (t', x', y', z') in S' Then, by eliminating the irrelevant frame times t & t' from the expressions for T & X, we get:

$$X = x/cosh\alpha - T\, tanh\alpha = x'/cosh\alpha' - T\, tanh\alpha'$$

Further, using $\omega = \alpha - \alpha' = -\omega'$ to eliminate α or α', we recover ***LT*** for the privileged time T:

(2a) $\quad \underline{x' = \{x\, cosh(\omega - \alpha) - T\, sinh\omega\}/cosh\alpha}$

(2b) $\quad \underline{x = \{x' cosh(\omega' - \alpha') - T\, sinh\omega'\}/cosh\alpha'}$

Finally, introducing non-standard frame-times $\bar{\tau}$ & $\bar{\tau}'$ for frames S & S' defined by means of :

(3) $\quad \underline{\bar{\tau} \equiv T/\gamma} \;\; . \;\; \underline{\bar{\tau}' \equiv T/\gamma'}$

$$\gamma \equiv cosh\alpha \equiv 1/\sqrt{1 - v^2} \;,\; \gamma' \equiv cosh\alpha' \equiv 1/\sqrt{1 - v'^2}$$

with $w \equiv tanh\omega$, we recover the Tangherlini transformations ***(TT)*** as generalized by Selleri:

(4a) $\quad x' = x\, \frac{cosh(\omega - \alpha)}{cosh\alpha} - \bar{\tau}\, sinh\omega = \frac{x\,(1 - wv) - w\bar{\tau}}{\sqrt{1 - w^2}}$

(4b) $\quad x = x' \frac{cosh(\omega' - \alpha')}{cosh\alpha'} - \bar{\tau}' sinh\omega' = \frac{x'(1 - w'v') - w'\bar{\tau}'}{\sqrt{1 - w'^2}}$

In standard *SR*, it is always the proper time of a single moving clock which is said to be retarded relative to the slave clocks distributed as a network over the rest frame of the observer; but if we refer the inertial motion of particles to a privileged frame we should use *TT* instead. Please notice that *TT* reduce to *GT* if all observations refer to the frame of the midway particle:

(5) $\quad \alpha = \frac{\omega}{2} \;\Rightarrow\; x - x' = \bar{\tau}\, sinh\omega = 2T sinh\frac{\omega}{2}$

Applying $\bar{\tau} \equiv t - x\, tanh\frac{\omega}{2} \equiv t' - x' tanh\frac{\omega'}{2} \equiv \bar{\tau}'$ directly to *LT* we get the same result, viz., *GT*:

(6) $\quad \underline{\bar{\tau}' = \bar{\tau} \;,\; \omega' = -\omega \;,\; x' = x - \bar{\tau}\, sinh\omega \;,\; y' = y \;,\; z' = z}$

3. THE ROBERTSON-WALKER METRIC

In his classic monograph *Natural Philosophy of Time* (1961/1980), G.J. Whitrow devised a method to deduce the ***RWM*** of relativistic standard cosmology from the γ-factor of ***SR***. Let:

$$c_o \equiv 1 \;\; . \;\; c_o t_o \equiv r_o \equiv 1$$

and let the *origo* of the comoving standard rest frame S of an observer P be P himself.

Now suppose an event E, taking place at some object O, to be triggered by a light signal which instantaneously released a visible flash. Suppose further that this light signal was sent off by P at the instant τ_1, and that the flash was received by P at the instant τ_3, both τ_1 & τ_3 being instants of proper time τ of P as read off his own standard atomic clock C. We then recover the Einstein coordinates of the Cartesian frame S of P by means of the usual definitions:

$$\tau_3 \equiv t{+}r \;\; . \;\; \tau_3' \equiv t'{+}r'$$

$$\tau_1 \equiv t{-}r \;\; . \;\; \tau_1' \equiv t'{-}r'$$

From the standard ***SR*** invariant $d\tau_3 d\tau_1$ we get the γ-factor for the retardation of moving clocks:

$$d\mathcal{T}^2 \equiv d\tau_3 d\tau_1 = dt^2{-}dr^2 \equiv dt^2\gamma^{-2}$$

Whitrow then suggested: $dr \rightarrow \mathcal{S}(T)\, d\sigma$. The parameter T of his function $\mathcal{S}(T)$ is thus no longer the *private frame time* t, but rather the *public proper time* τ of fundamental observers, i.e., all observers at rest in the universe, e.g., relative to the cosmic background radiation ***(CBR)***. With $T \equiv \tau$, the standard invariant of ***SR*** is finally transformed into the standard ***RWM*** metric:

$$d\mathcal{T}^2 = \; dt^2 - dr^2 = d\tau^2 - \mathcal{S}^2(\tau)\, d\sigma^2$$

Here $\mathcal{S}$ is taken to be the *universal scale factor* and σ a fixed ("comoving") coordinate. Now, for *fundamental observers* $d\sigma = 0$, i.e., $d\mathcal{T} = d\tau$, showing that all fundamental observers are in pace with the same common *cosmic time* $\mathcal{T}$. By implication, any deviation of proper time τ from $\mathcal{T}$ is confined to non-fundamental, i.e., *accidental observers* discerned by a variable σ. Considering $\mathcal{T} \neq \tau \neq t$, one may ask if all this amounts to more than a vague analogy?

According to the standard view, it is always the *proper time* of a "moving" particle that is claimed to be "slow" relative to the *frame time* of a "stationary" observer. So coordinate time, or frame time, is thereby tacitly assumed to represent the "true extended time" of any observer. The cosmic time $\mathcal{T}$ implied by ***RWM*** is seldom taken seriously, but mostly ignored or explained away as being of "statistical origin" and thus "ill defined". Nevertheless, it is the firm stance of the present writer that a fundamental importance should be ascribed to the cosmic time $\mathcal{T}$.

If we define *true time* by the readings of the master clocks of our fundamental observers when they have been properly synchronized - e.g., by letting a definite non-local cosmic event, such as the beginning of everything in a socalled "big bang", represent a common time zero - then it is no longer true to say that the master clock of a fundamental observer is slow relative to the frame clocks of another observer, fundamental or not. Much rather it is true to say that it is the clocks of accidental particles that are slow relative to the clocks of fundamental observers. But the only conflict at stake here is one relating to the standard interpretation of ***SR***.

Hence, if the clocks of fundamental observers show the true time $\mathcal{T}$, then the clock of an accidental particle will be more or less slow. In fact, *the greater its distance to that fundamental particle relative to which it is momentarily at rest, and which thus constitutes the natural origo of its own rest frame, the slower its clock will run* and the more it will deviate from true time. The natural way of interpreting this retardation of moving clocks is as an effect of gravitation.

In this way we have found a simple coupling between the rates of non-fundamental clocks and what seems to be a gravitational potential due to the substratum of fundamental particles.

The reason for this dependence is that *the deviation of the clock of an accidental particle from true time $\mathcal{T}$ is found by direct comparison with the clock of that fundamental particle with which it momentarily coincides*; and the greater the distance of an accidental particle is to the origo of that rest frame to which it belongs, the faster its speed relative to that fundamental particle with which it coincides will appear; this follows from the expansion function $\mathcal{S}(\tau)$. What we have disclosed is the possibility of an influence of the substratum on particles which do not belong to the substratum and which represent deviations from cosmic symmetry.

This supports the stance of Whitrow's former master, E.A. Milne. The essential point of his ***Kinematic Relativity*** (***KR***), devised as an alternative to Einstein's theories, ***SR*** & ***GR***, is that what we call gravitational effects may be due to local deviations from cosmic symmetry. Indeed, if elevated to a universal principle, *Milne's conjecture amounts to nothing less than an inversion of Mach's principle: while Mach held that inertia should be reduced to gravitation, Milne insisted that gravitation should be reduced to inertia* - and showed how to do it!

Now, with polar coordinates, the ***RWM*** can be written:

(6) $$\underline{d\mathcal{T}^2 = d\tau^2 - \mathcal{S}^2(\tau)\{d\rho^2+\lambda^2(d\theta^2+sin^2\theta\, d\phi^2)\}}$$

(7) $$\underline{\mathcal{R} \equiv \mathcal{S}(\tau)\rho\ ,\ \mathrm{T} \equiv \int d\tau/\mathcal{S}(\tau) + const.\,,\ \mathcal{C}(\mathrm{T}) \equiv \mathrm{dT}/d\tau \equiv 1/\mathcal{S}(\tau)}$$

Here $\mathcal{R}$ is proper distance, $\mathcal{C}$ is an inverse scale function, and T is an auxiliary time parameter.

$$d\rho \equiv d\lambda/\sqrt{1-\kappa\lambda^2} = \begin{cases} d\lambda & \Leftarrow \kappa = 0 \\ d\,arcsin\lambda & \Leftarrow \kappa = 1 \\ d\,arsinh\lambda & \Leftarrow \kappa = -1 \end{cases}$$

$$d\mathcal{T}^2 \underset{\kappa=0}{\equiv} d\tau^2 - \mathcal{S}^2(\tau)\{d\rho^2 + \rho^2(d\theta^2+sin^2\theta\, d\phi^2)\}$$

$$d\mathcal{T}^2 \underset{\kappa=1}{\equiv} d\tau^2 - \mathcal{S}^2(\tau)\{d\rho^2 + sin^2\rho(d\theta^2+sin^2\theta\, d\phi^2)\}$$

$$d\mathcal{T}^2 \underset{\kappa=-1}{\equiv} d\tau^2 - \mathcal{S}^2(\tau)\{d\rho^2 + sinh^2\rho(d\theta^2+sin^2\theta\, d\phi^2)\}$$

$$d\rho \equiv d\lambda/\sqrt{1-\kappa\lambda^2} \equiv d\varrho/(1+\kappa\varrho^2/4) = \begin{cases} d\varrho & \Leftarrow \kappa = 0 \\ d\,arctan(\varrho/2) & \Leftarrow \kappa = 1 \\ d\,artanh(\varrho/2) & \Leftarrow \kappa = -1 \end{cases}$$

$$d\rho^2 + \lambda^2(d\theta^2+sin^2\theta\, d\phi^2) \equiv \frac{d\varrho^2+\varrho^2(d\theta^2+sin^2\theta\, d\phi^2)}{1+\kappa\varrho^2/4} \equiv \frac{d\xi^2+d\eta^2+d\zeta^2}{1+\kappa\varrho^2/4}$$

The expressions below cover all possible values of the constant of curvature, κ:

(8a) $$\underline{d\mathcal{T}^2 = d\tau^2 - \mathcal{S}^2(\tau)\{d\lambda^2/(1-\kappa\lambda^2) + \lambda^2(d\theta^2+sin^2\theta\, d\phi^2)\}}$$

(9b) $$\underline{= d\tau^2 - \mathcal{S}^2(\tau)\{d\varrho^2 + \varrho^2(d\theta^2+sin^2\theta\, d\phi^2)\}/(1+\kappa\tfrac{\varrho^2}{4})}$$

(9c) $$\underline{= \mathcal{C}^{-2}(\mathrm{T})[\,\mathrm{dT}^2 - \{d\xi^2+d\eta^2+d\zeta^2\}/(1+\kappa\tfrac{\varrho^2}{4})]}$$

Applying the T-scale, the *expansion of cosmos* is explained away as a *shrinking of atoms*! In standard presentations, great importance is generally ascribed to the Hubble function $\mathcal{H}$:

$$\underline{\mathcal{H}(\tau) \equiv \dot{\mathcal{S}}(\tau)/\mathcal{S}(\tau)}$$

4. MILNE'S SIMPLE BIG BANG MODEL

In what follows we throw light on the ***RWM*** by discussing some simple world models. One of the simplest is Milne's model of uniform expansion, adopted by Prokhovnik [1967]:

(9) $$\underline{\mathcal{S}(\tau) \equiv \tau/\tau_o \equiv d\tau/d\mathrm{T} \equiv e^{(\mathrm{T}-\tau_o)/\tau_o} \equiv \mathcal{C}^{-1}(\mathrm{T})}$$
$$\underline{\mathcal{H}(\tau) \equiv \dot{\mathcal{S}}(\tau)/\mathcal{S}(\tau) \propto 1/\tau}$$

Let us assume that radar signals are being exchanged between a pair of observers, P & Q, in $(1\mathrm{x}1)$ *timespace*. Suppose that a "photon" ϕ is emitted from P at $\tau = \tau_1^p$, reflected by Q at $\tau = \tau_2^q$, and received by P at $\tau = \tau_3^p$. Then, according to the relativity principle as interpreted by Milne, τ_3^p is the same function of τ_2^q as τ_2^q is of τ_1^p - call it $s(\tau) \equiv e^{\sigma}\tau$. Generalizing, and introducing Einsteinian standard coordinates t & r for P (priming those of Q), we at once get:

$$t \equiv \tfrac{1}{2}(\tau_3+\tau_1) \;\Downarrow\; r \equiv \tfrac{1}{2}(\tau_3-\tau_1)$$
$$\tau_3 = \tau\, e^{\sigma} = t + r \;\;.\;\; \tau_1 = \tau\, e^{-\sigma} = t - r$$
(10) $$\underline{t = \tau\, cosh\sigma} \;.\; \underline{r = \tau\, sinh\sigma}$$

Let us next assume that σ is not a constant, but a variable; then, by differentiation:

$$dt = d\tau\, cosh\sigma + \tau\, d\sigma\, sinh\sigma$$
$$dr = d\tau\, sinh\sigma + \tau\, d\sigma\, cosh\sigma$$
$$d\mathcal{T}^2 \equiv dt^2 - dr^2 = d\tau^2 - \tau^2 d\sigma^2 = e^{2(\mathrm{T}-\tau_o)/\tau_o}(d\mathrm{T}^2 - d\sigma^2)$$

This invariant is easily expanded into a hyperbolic *timespace* of $(1\mathrm{x}3)$ dimensions if we put:

$$d\sigma^2 \equiv d\rho^2 + sinh^2\rho\,(d\theta^2 + sin^2\theta\, d\phi^2) \equiv \{d\xi^2 + d\eta^2 + d\zeta^2\}/(1 - \tfrac{\varrho^2}{4})$$

The standard ***SR*** invariant is thus transmuted into the hyperbolic metric of an expanding universe with expansion function $\mathcal{S}(\tau) \equiv \tau$, which can be transformed into another hyperbolic metric, viz., that of a stationary universe whose atoms all contract in agreement with the Hubble function $\mathcal{C}^{-1}(\mathrm{T}) \equiv e^{(\mathrm{T}-\tau_o)/\tau_o}$, where $\mathrm{T} = \tau_o\{1 + log(\tau/\tau_o)\}$, τ_o being a constant of calibration:

(11a) $$\underline{d\mathcal{T}^2 = dt^2 - dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2)}$$

(12b) $$\underline{= d\tau^2 - \tau^2\{d\rho^2 + sinh^2\rho\,(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

(12c) $$\underset{\tau_o \equiv 1}{\equiv} \underline{e^{2(\mathrm{T}-1)}[\,d\mathrm{T}^2 - \{d\xi^2 + d\eta^2 + d\zeta^2\}/(1 - \tfrac{\varrho^2}{4})]}$$

The **1**st of these metrics, incorporating the universal constancy of the velocity of light, yields an infinity of *private timespaces*, comprising the flat 3-spaces of fundamental observers. The following two metrics both yield *a public timespace*, each containing a curved 3-space: that of the **2**nd metric being associated with τ-*time*, relative to which atoms keep constant sizes while the distances between fundamental observers steadily expand in proportion to $\mathcal{S} \equiv \tau$, with the consequence that *light is stretched*, as propsed by Prokhovnik - and that of the **3**rd metric being associated with T-*time*, relative to which distances between fundamental observers remain invariant whereas the sizes of their atomic constituents are contracting in proportion to $\mathcal{C}^{-1} \underset{\tau_o=1}{\equiv} e^{\mathrm{T}-1}$, due to *a secular reduction* of the velocity of light, as explained by Whitrow.

5. THE FIRST STEADY STATE MODEL

Now, passing from Milne's world model to that of Gold & Bondi, and of Hoyle, the scale factor $\mathcal{S}(\tau)$ is changed from $\tau \equiv \tau/\tau_o$ to $e^{\tau} \equiv e^{\tau/\tau_o}$, characterizing the "steady state" model: so:

(12)
$$\underline{\mathcal{S}(\tau) \equiv e^{\tau} \equiv d\tau/\mathrm{dT} \equiv \tfrac{1}{1-\mathrm{T}} \equiv \mathcal{C}^{-1}(\mathrm{T})}$$
$$\underline{\mathcal{H}(\tau) \equiv \dot{\mathcal{S}}(\tau)/\mathcal{S}(\tau) \propto const.}$$

$\mathcal{R} \equiv e^{\tau}\rho \equiv tanh\, r$ is a candidate to the proper distance between fundamental particles, just as $e^{t-\tau} \equiv 1/\sqrt{1-\mathcal{R}^2} = cosh\, r$ is a plausible relationship of frame time t to proper time τ. Hence, if Bondi & Gold, and Hoyle, want to retain $d\tau^2 - e^{2\tau}d\rho^2$ as a fundamental invariant of their model, in face of the definitions $\rho \equiv sinh\, r\, e^{-t} \equiv tanh\, r\, e^{-\tau}$, they shall have to accept:

(13)
$$d\tau = dt - dr\, tanhr \;\; \Downarrow \;\; e^{\tau}d\rho = dr - dt\, tanhr \;\; *$$
$$d\mathcal{T}^2 = d\tau^2 - e^{2\tau}d\rho^2 = \{\tfrac{\mathrm{dT}^2 - d\rho^2}{(1-\mathrm{T})^2}\} = \tfrac{dt^2 - dr^2}{cosh^2 r}$$

Generalizing these to (1x3) dimensional timespace we find the following three metrics, of which the first one comprises the public flat 3-space of a universe expanding with $\mathcal{S}(\tau) = e^{\tau}$, whereas the second one contains the public flat 3-space of atoms shrinking with $\mathcal{C}(\mathrm{T}) = 1-\mathrm{T}$, and the third one is closest to represent the private 3-spaces of the standard frames of ***SR***:

(14a)
$$\underline{d\mathcal{T}^2 \;=\; d\tau^2 - e^{2\tau}\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

(14b)
$$\underline{=\tfrac{1}{(1-\mathrm{T})^2}[\,\mathrm{dT}^2 - \{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}]}$$

(14c)
$$\underline{= [\,dt^2 - \{dr^2 + sinh^2 r(d\theta^2 + sin^2\theta\, d\phi^2)\}]\,\tfrac{1}{cosh^2 r}}$$

$d\mathcal{T}^2 = [\,dt^2 - \{dr^2 + sinh^2 r(d\theta^2 + sin^2\theta\, d\phi^2)\}]/cosh^2 r$ does not compete well with the standard invariant of ***SR*** which is the much simpler one: $d\mathcal{T}^2 = dt^2 - dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2)$. This rather serious problem is caused by the external factor $cosh^{-2}r$ which is much less akin to the ***LT*** of ***SR*** than to the ***VT*** (Voigt transformations) of some competing aether theory.

Therefore, it seems worth while to search for alternative "steady state" models that are not at variance with $d\mathcal{T}^2 = dt^2 - dr^2$. As a step on the way, I shall propose the model below where (15b) & (15c) follow from (15a) by the above definitions $\rho \equiv sinh\, r\, e^{-t} \equiv tanh\, r\, e^{-\tau}$:

(15a)
$$d\mathcal{T}^2 \;=\; dt^2 - \{dr^2 + sinh^2 r(d\theta^2 + sin^2\theta\, d\phi^2)\}$$

(15b)
$$= [\,d\tau^2 - e^{2\tau}\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}]\,\tfrac{1}{1-e^{2\tau}\rho^2}$$

(15c)
$$= \tfrac{1}{(1-\mathrm{T})^2}[\,\mathrm{dT}^2 - \{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}]\,\tfrac{1}{1-\rho^2/(1-\mathrm{T})^2}$$

This, at the very least, is compatible with the standard invariant $d\mathcal{T}^2 = dt^2 - dr^2$ of ***SR***. But $d\tau$ & dT do not represent cosmic time since an external factor applies to both (16b) & (16c). This clearly shows that (15b) & (15c) do not conform to the ***RWM*** for expanding space.

* $$e^{\tau}d\rho = dr\, cosh^{-2}r - d\tau\, tanh\, r = dr\tfrac{(1+sinh^2 r)}{cosh^2 r} - dt\, tanh\, r = dr - dt\, tanh\, r$$

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6. A NEW STEADY STATE MODEL

Let us make a fresh start by adopting $d\mathcal{T}^2 = dt^2 - dr^2$ of ***SR***. As we need not accept that standard frames are flat, it seems that we are free to assume that the world is better described, when referring to frame time t, by assuming the 3-space of standard frames to be hyperbolic:

(16) $$\underline{d\mathcal{T}^2 \equiv dt^2 - \{dr^2 + sinh^2 r\,(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

Now the shortcomings alluded to in §5 can be remedied by adopting the following definitions:

(17) $$\underline{\rho \equiv sinh\, r\, e^{-t} \equiv 2\, tanh\tfrac{r}{2}\, e^{-\tau} \equiv [2]\mathcal{R}e^{-\tau}}$$

Interpreting $\mathcal{R}$ as twice the distance to *the midway particle* between two fundamental observers, we shall choose $\mathcal{R} \equiv 2\, tanh\frac{r}{2} \to 2$ rather than $\mathcal{R} \equiv tanh\frac{r}{2} \to 1$, deleting the factor $[2]$ in (17).

From the above definitions we immediately derive the relationships following below:

(18) $$\underline{\mathcal{R} = e^{\tau}\rho \;\Rightarrow\; \dot{\mathcal{R}}/\mathcal{R}}$$

(19) $$\underline{e^t d\rho = dr\, cosh\, r - dt\, sinhr = dr - d\tau\, sinh\, r}$$

For fundamental observers, $d\rho = 0$; hence $v \equiv dr/dt = tanhr$, $w \equiv dr/d\tau = sinhr$, whence:

(20) $$\underline{cosh\, r = \tfrac{1}{\sqrt{1-v^2}} = \gamma_r} \;.\; \underline{sinhr = \tfrac{v}{\sqrt{1-v^2}} = v\,\gamma_r}$$

As shown in the preceding section, the definitions $\rho \equiv sinh\, r\, e^{-t} \equiv tanh\, r\, e^{-\tau}$ might be used to change the old steady state metric (14*a*), $d\mathcal{T}^2 = d\tau^2 - e^{2\tau}\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}$, into two other metrics (14*b*) & (14*c*) where neither τ, nor T, were apt to represent a cosmic time; the third metric was furthermore marred by an external factor that prevented a reduction to ***SR***. Fortunately, the alternative definitions $\rho \equiv sinh\, r\, e^{-t} \equiv 2\, tanh\frac{r}{2}\, e^{-\tau}$ work much better since, as we have just seen, they allow us to preserve the γ-factor, so characteristic for ***SR***.

It is also evident that for $d\theta = d\phi = 0$ (16) reduces to $d\mathcal{T}^2 = dt^2 - dr^2 = dt^2/\gamma^2 = d\tau^2$. Thus we have tested our basic assumption, viz., that the master clocks of fundamental particles always keep the same *Cosmic Rhythm*, $d\mathcal{T} = d\tau =$ *invar*., the core of a *Cosmic Time*.

Note added 2021:

So far, I have been unable to devise a plausible ***RW***-compatible metric for expanding space:

$$e^{t-\tau} = cosh^2\tfrac{r}{2} \;\Downarrow\; e^{\tau}\rho = 2\, tanh\tfrac{r}{2}$$

$$e^t d\rho = dr\, cosh\, r - dt\, sinhr = dr - d\tau\, sinh\, r$$

$$dr = d\tau\, sinh\, r + e^t d\rho = (d\tau\, 2\, tanh\tfrac{r}{2} + e^{\tau} d\rho)\, cosh^2\tfrac{r}{2} = \frac{e^{\tau}\rho\,(d\tau + \frac{d\rho}{\rho})}{1 - e^{2\tau}\rho^2}$$

$$dt = d\tau + dr\, tanh\tfrac{r}{2} = d\tau + dr\tfrac{1}{2}e^{\tau}\rho = \frac{d\tau\,(1 - e^{2\tau}\rho^2) + \frac{1}{2}e^{2\tau}\rho^2\,(d\tau + \frac{d\rho}{\rho})}{1 - e^{2\tau}\rho^2} = \frac{d\tau\{1 - \frac{1}{2}e^{2\tau}\rho^2(1 - \frac{d\rho}{\rho\, d\tau})\}}{1 - e^{2\tau}\rho^2}$$

$$d\mathcal{T}^2 = dt^2 - dr^2 = (dt + dr)(dt - dr) =$$

$$\left\{\frac{d\tau\{1 - \frac{1}{2}e^{2\tau}\rho^2(1 - \frac{d\rho}{\rho\, d\tau})\}}{1 - e^{2\tau}\rho^2} + \frac{d\tau e^{\tau}\rho\,(1 + \frac{d\rho}{\rho\, d\tau})}{1 - e^{2\tau}\rho^2}\right\}\left\{\frac{d\tau\{1 - \frac{1}{2}e^{2\tau}\rho^2(1 - \frac{d\rho}{\rho\, d\tau})\}}{1 - e^{2\tau}\rho^2} - \frac{d\tau e^{\tau}\rho\,(1 + \frac{d\rho}{\rho\, d\tau})}{1 - e^{2\tau}\rho^2}\right\}$$

$$= \frac{d\tau^2\left[\{1 - \frac{1}{2}e^{2\tau}\rho^2(1 - \frac{d\rho}{\rho\, d\tau})\}^2 - e^{2\tau}\rho^2(1 + \frac{d\rho}{\rho\, d\tau})^2\right]}{(1 - e^{2\tau}\rho^2)^2} \underset{-d\rho = \rho\, d\tau}{\equiv} d\tau^2$$

That $dr = 0$ for $d\rho = -\rho\, d\tau$ means that we follow a series of receding FPs crossing a fixed distance r_o.

Therefore the 3-space of my new world-model is not expanding, but fixed, i.e., stationary.

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7. CONCLUSION

A very simple alternative construction of my new model, ignoring important details, since the metric is only valid for fundmental particles, might run as follows, writing *c = unity*, using $t \equiv \frac{1}{2}(t_3{+}t_1)$ & $r \equiv \frac{1}{2}(t_3{-}t_1)$ for *frame-time* & *frame-distance*, resp., and postulating:

$$\underline{dt/d\mathcal{T} \ = cosh(r/r_o)} \ \ . \ \ \underline{dr/d\mathcal{T} \ = sinh(r/r_o)}$$

From these two simple differential equations all important consequences follow in due order:

$$d\mathcal{T}^2 = dt^2{-}dr^2 \ \ . \ \ v \equiv dr/dt \ = tanh(r/r_o)$$
$$dt/d\mathcal{T} \ = 1/\sqrt{1{-}v^2} \equiv \gamma \ \ . \ \ dr/d\mathcal{T} \ = v/\sqrt{1{-}v^2} = v\,\gamma$$
$$1{+}z(t) \equiv (dt{+}dr)/d\mathcal{T} = exp\,\{r(t)/r_o\} = d\mathcal{T}/(dt{-}dr)$$
$$1{+}z(t) = exp\,(r/r_o) = e \ \Leftrightarrow \ r = r_o \equiv t_o \equiv unity$$
$$\mathcal{R} \equiv 2\,tanh(\tfrac{r}{2}) \ \Rightarrow \ \dot{\mathcal{R}} \equiv d\mathcal{R}/d\mathcal{T} \propto \mathcal{R}$$
$$\underline{\mathcal{H} \ = \dot{\mathcal{R}}(\tau)/\mathcal{R}(\tau) = const.}$$

From the Hubble proportionality $\mathcal{H}$, velocity-space being hyperbolic (see Ungar 2008), it is natural to conclude that position-space must be hyperbolic too; so we shall postulate:

$$\underline{d\mathcal{T}^2 \ = \ dt^2 - dr^2 - sinh^2 r\,(d\theta^2{+}sin^2\theta\,d\phi^2)}$$

Here, $d\mathcal{T}^{-1}$ may be interpreted as representing an all-embracing ***cosmic rhythm***, and we then only have to agree about the arbitrary choice of an universal time *zero*, in order possess a full-fledged ***cosmic time*** overcoming the reservations expressed by Bondi [1959^2, p.70].

A feature of my new metric is that the element $d\mathcal{T}$ of timespace can be interpreted as the fundamental element of a *cosmic supertime*, as suggested by my mentor André Mercier.

What is also new as compared to the old model is that our metric obeys to the *no-horizon postulate* of Milne; so it survives the number-redshift statistics that refuted the old one.

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